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Bernstein Polynomials: a Bibliometric Data Analysis Since the Year 1949 Based on the Scopus Database[1]

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Abstract

It's hard to imagine human life in the digital and AI age without polynomials, as they are everywhere but mostly invisible to ordinary people: in data trends, on computer screens, in the shapes around us, and in the very fabric of technology. One of these, the simple but elegant Bernstein polynomials, was discovered by a scientist from the Russian Empire, Sergei Bernstein, in 1912 and plays a central role in mathematical analysis, computational and applied mathematics, geometric modelling, computer-aided geometric design, computer graphics, and other areas of science and engineering. They have been the sub-ject of much research for over a hundred years. However, no work has carried out database-derived research analysis, such as bibliometric, keyword, or network analysis, or, more generally, data analysis of manuscript data related to Bernstein polynomi-als extracted from digital academic databases. This work, which appears to be the first-ever attempt at bibliometric data anal-ysis of Bernstein polynomials, aims to fill this gap and open researchers' eyes to potentially new or underexplored areas of mathematics and engineering where Bernstein polynomials may one day be used to make discoveries. The results may be helpful to academics researching Bernstein polynomials and looking for potential applications, collaborators, supervisors, funding, or journals to publish in.

Keywords: Sergei Bernstein; Bernstein polynomial; Scopus database; keywords; bibliometric analysis; data analytics; geometric model-ling; CAGD; CAD; approximation; network analysis; visualisation.

"Today all mathematicians and physicists agree that the field of applications for mathematics knows no limits except those of knowledge itself."
Sergei Bernstein

1. Introduction

It's hard to imagine human life in the digital and AI age without polynomials, as they are everywhere around us but often invisible to most people: in data trends, on computer screens, in the shapes around us, and the very fabric of technology. Polynomials drive computer animation in movies, optimise routes in navigation systems such as GPS, model complex physical phenomena from planetary motion to the spread of infectious diseases, and are part of models of many complex systems that help us better understand the world around us. They're at the heart of the algorithms that process images, the central part of the geometric kernels in CAD systems. They're used to predict weather conditions and compress the music we listen to daily. In engineering, they define structural designs and stress analysis; in economics, they model growth and fluctuations; in machine learning, they help build predictive models. Polynomials are not just equations made up of mathematical symbols - they are the silent archi-

[1] The paper is available online as a preprint at https://arxiv.org/abs/2503.07614.

tects of the modern digital world, shaping how we interact with technology, interpret data, and design innovation contours.

One of these polynomials is the simple but elegant Bernstein polynomial, which has been a pivotal concept in mathematics since it was introduced by Sergei Bernstein in 1912, and later found its applications in the fields of computer-aided geometric design (CAGD) and computer-aided design (CAD) [1]. These polynomials have the following form

$$B_n(x) = \sum_{\nu=0}^{n} \beta_\nu b_{\nu,n}(x)$$

where β_ν is a Bernstein coefficient, and $(n+1)$ Bernstein basis polynomials of degree n are defined by

$$b_{\nu,n}(x) = \binom{n}{\nu} x^\nu (1-x)^{n-\nu}, \nu = 0, \dots, n,$$

where $\binom{n}{\nu}$ is a binomial coefficient.

Initially used in a constructive proof for the Weierstrass approximation theorem, these simple but elegant polynomials have evolved into a fundamental tool for approximation theory, numerical analysis, and computer-aided geometric design (CAGD) [1]. Their simple yet profound nature has allowed them to remain an active research topic in CAGD for more than one hundred years, stimulating further exploration and various applications in engineering.

The enduring significance of Bernstein polynomials stems from their simplicity, elegance, and versatility. Their key properties, such as non-negativity, partition of unity, and shape-preserving characteristics, make them indispensable for mathematical modelling, curve fitting, and optimisation problems [2]. Their association with Bézier curves and surfaces in the form of basis functions has also made them a cornerstone in computer graphics, computer-aided design (CAD), and VR, enabling the creation of visually appealing and mathematically precise geometric shapes vital for digital manufacturing and 3D printing.

Out of a total of 16,392 Scopus-indexed documents containing the keywords "Bézier surface" or "Bézier surface", "Bernstein-Bézier curve" or "Bernstein-Bézier surface" are only mentioned in 416 documents in all search fields, i.e. only 2.57%. From one point of view, this is a pity, but from another point of view, these days, who mentions the software or programming language used for their discoveries in the title of a manuscript or the name of a new theorem or method? The various tools we use in research have often become the silent architects of scientific progress.

The applications of Bernstein polynomials and their generalisations have expanded across diverse disciplines, including recognition of human speech [3], probability [4], and the theory of desirable gambles [5]. This growth has been accompanied by extensive research, continuously enhancing our understanding of these polynomials and their properties and reflecting their ongoing relevance. Over the past century, the research on Bernstein polynomials has experienced significant growth and development [6].

This manuscript presents a pioneering bibliometric data analysis of Bernstein polynomials based on RIS data file download from the Scopus database (http://scopus.com/), analysing publications since 1949. The aim is to examine the evolution of research topics, identify key contributors and organisations, funding agencies, and influential works, and search for areas where Bernstein polynomials are rarely mentioned. This analysis aims to provide a valuable resource that encapsulates Bernstein polynomials' historical progress and future potential. The Scopus search query TITLE-ABS-KEY(polynomial AND "bibliometric analysis") returns 23 papers, but "polynomial" is not in the title of these papers, which means that our work may be the first-ever attempt at bibliometric analysis of polynomials, at least based on papers indexed in the Scopus database.[2] of polynomials, at least based on papers indexed in the Scopus database.

[2] Authors should not confuse ***bibliometric analysis*** with ***bibliographic analysis***, which are related but distinct concepts in the study of academic literature and research output.

2. Materials and Methods

2.1. Data Collection and Processing

A search was conducted for documents (excluding preprints, patents, and secondary documents) with the keyword "Bernstein polynomial" in all fields. Scopus, one of the largest web bibliometric databases, was used to obtain the dataset. Several keywords were used to find weak connections to Bernstein polynomials and to attract academics for research on Bernstein polynomials applied to found areas. Our study excludes documents that may use Bernstein polynomials under other names, as they were called before the term "Bernstein polynomials" became widely accepted in the academic community.

2.2. Analysis Procedure and Software

To carry out a bibliometric analysis, we followed the generally accepted steps: (1) study or research design, (2) data collection, (3) data analysis, (4) data visualisation, and (5) interpretation of results. We used VOSviewer v. 1.6.20 software and various AI chatbots such as ChatGPT, Llama, Mixtral, and Perplexity to analyse and visualise our data. The simple statistics, including the most productive authors, organisations, and others, were extracted from the Scopus database, and the relative percentages were calculated in Microsoft Excel. To carry out the co-citation analysis, we used the VOSviewer[3] software, which is used in bibliometric analysis to map networks and evaluate the strength of links.

We used the standard 32-bit colour maps available in VOSviewer to visualise the networks. The colour maps used facilitate the presentation of large amounts of bibliometric data and allow clear visualisation of network structures. However, users can easily customise and extend this colour mapping functionality by designing their colour maps for more accurate colour representation. This flexibility allows the creation of more nuanced and detailed visualisations, particularly for huge network datasets.

When exporting visualisations in VOSviewer, the software adds its logo in the lower left corner, but it's missing on screenshots. Again, we want to emphasise that all visualisations have been made with the software mentioned. As no copyright information exists on their website, adding the logo could be seen as an advertising method.

3. Basic Statistics from Scopus Database

Basic statistics, such as subject area, source title, etc., can be copied or scraped directly from the Scopus search results page. As of 23 January 2025, the keyword "Bernstein polynomial" is mentioned in 2371 documents' titles, abstracts or keywords, and the search within all fields returns 9310 documents starting from 1949. Such a significant difference may indicate that many papers cite "Bernstein polynomials" without directly researching them but using them in various fields. As we are looking for multiple applications of these polynomials, we would consider the dataset with 9310 documents, including 77 articles in press, for our research. Of these, 84.9% are articles, 11% conference papers, 1.8% book chapters, 1.1% reviews, 0.8% books, 0.2% notes, 0.1% editorials and 0.3% other sources. Interestingly, only two retracted papers among the documents show a high academic culture level in this field.

Most of the Scopus subject areas that mention the Bernstein polynomial are shown in Figure 1, where the most popular subject areas are Mathematics, Computer Science, Engineering, and Physics & Astronomy. Fields such as Materials Science, Chemistry, Earth and Planetary Sciences, Energy, Biochemistry, Genetics and Molecular Biology, Agricultural and Biological Sciences, Multidisciplinary, Environmental Science, Economics, Econometrics and Finance, Social Sciences, Chemical Engineering, Medicine, Business, Management and Accounting, Neuroscience, Immunology and Microbiology, Arts and Humanities, Health Profes-

[3] https://www.vosviewer.com/

sions, Pharmacology, Toxicology and Pharmaceutics, Psychology, Nursing, Dentistry have less than 2% of all documents for each field.

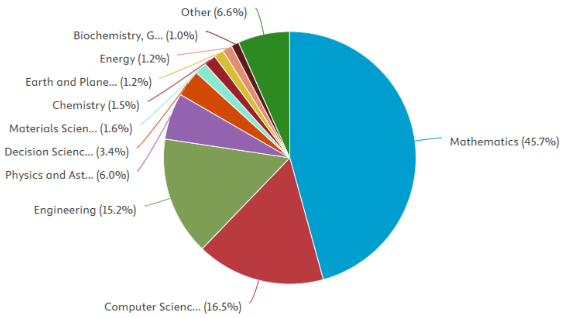
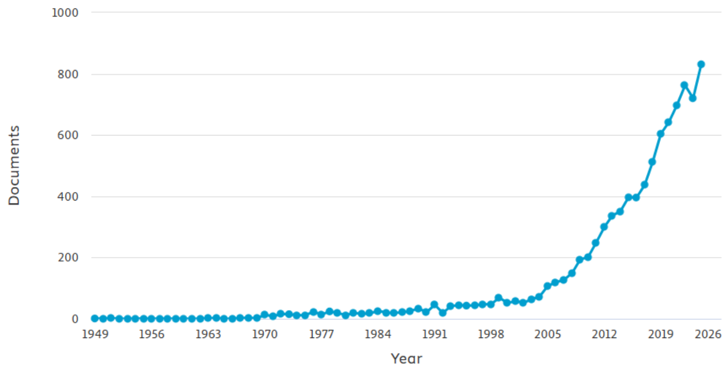
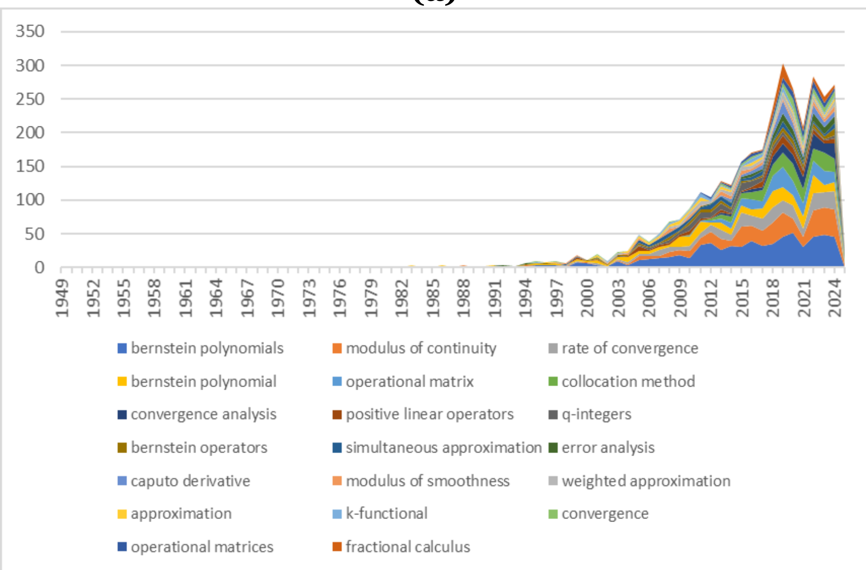


Figure 1. A pie chart from Scopus shows the percentage of total mentions of the keyword "Bernstein polynomial" in different subject areas.

Figure 2 shows the dynamics of annual publications and the twenty most popular keywords since 1949. From around 2005, we can observe a significant growth in the number of documents and, consequently, in the number of keywords.



(a)



(b)

Figure 2. (a) Number of documents since 1949. The year 2025 is not shown here, and 70 publications have been indexed in Scopus. (b) The evolution of the twenty most popular keywords in manuscripts. The evolution of keywords with ten or more occurrences, as produced by the ScienceScape[4] online tool, can be found online at https://youtu.be/DQBZHmngCNo

[4] https://medialab.github.io/sciencescape/

The 100 most popular sources, mainly containing prestigious journals, account for 4060 or 43.6% of the 9310 documents in the database (see Table 1). Most of these are mathematics, computer, and applied sciences-related periodicals.

Table 1. The list of the most popular source titles where the keyword "Bernstein polynomial" appears in all search fields.

Source name	Number of documents
Journal of Approximation Theory	240
Applied Mathematics and Computation	205
Journal of Computational and Applied Mathematics	165
Mathematical Methods in the Applied Sciences	138
Journal of Inequalities and Applications	112
Journal of Mathematical Analysis and Applications	105
Filomat	101
Computer Aided Geometric Design	97
Mathematics (MDPI)	95
Journal of Computational Analysis and Applications	94
Advances in Difference Equations	91
Results in Mathematics	89
Lecture Notes in Computer Science, including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics	78
AIP Conference Proceedings	76
Symmetry	66
Abstract and Applied Analysis	61
Computational and Applied Mathematics	61
International Journal of Applied and Computational Mathematics	55
Computers and Mathematics with Applications	54
Numerical Algorithms	54
Advanced Studies in Contemporary Mathematics Kyungshang	51
Numerical Functional Analysis and Optimization	46
Mediterranean Journal of Mathematics	44
Applied Numerical Mathematics	42
Chaos Solitons and Fractals	40
Computational Statistics and Data Analysis	40
Proceedings of The Jangjeon Mathematical Society	40
Mathematics and Computers in Simulation	39
Demonstratio Mathematica	36
Aims Mathematics	35
Applied Mathematical Sciences	35
Acta Mathematica Hungarica	34
Constructive Approximation	33
Iranian Journal of Science and Technology Transaction A Science	33
Carpathian Journal of Mathematics	32
Springer Proceedings in Mathematics and Statistics	32
Fractal and Fractional	31
Engineering with Computers	30
Advances in Computational Mathematics	28
Alexandria Engineering Journal	28
Applied Mathematical Modelling	28
IEEE Transactions on Antennas and Propagation	28
Applied Mathematics Letters	27
Communications in Statistics Theory and Methods	27
IEEE Access	27

International Journal of Mathematical Analysis	27
Journal of Nonparametric Statistics	27
Turkish Journal of Mathematics	27
Applied Mathematics and Information Sciences	26
Approximation Theory and Its Applications	26
Journal of Physics Conference Series	26
Mathematical Sciences	26
Revista De La Real Academia De Ciencias Exactas Fisicas Y Naturales Serie A Matematicas	26
Journal of Function Spaces	25
Mathematical Foundations of Computing	25
International Journal of Computer Mathematics	24
Journal of Vibration and Control	24
Journal of Statistical Planning and Inference	24
Springer Optimization and Its Applications	24
Computational Methods for Differential Equations	23
Computer Methods in Applied Mechanics and Engineering	23
Journal of Mathematical Inequalities	23
Soft Computing	23
Calcolo	22
Fuzzy Sets and Systems	22
IEEE Transactions on Signal Processing	22
Journal of Multivariate Analysis	22
Positivity	22
International Journal of Pure and Applied Mathematics	21
Mathematical and Computer Modelling	21
Mathematical Problems in Engineering	21
Rocky Mountain Journal of Mathematics	21
Statistics and Probability Letters	21
Studia Scientiarum Mathematicarum Hungarica	21
Applied Mathematics	20
CMES Computer Modeling in Engineering and Sciences	20
Electronic Journal of Statistics	20
International Journal of Mathematics and Mathematical Sciences	20
Numerical Methods for Partial Differential Equations	20
Hacettepe Journal of Mathematics and Statistics	19
IEEE Signal Processing Letters	19
Journal of Computational Physics	19
Journal of The American Statistical Association	19
Advanced Studies in Theoretical Physics	18
Communications in Statistics Simulation and Computation	18
Georgian Mathematical Journal	18
Journal of Numerical Analysis and Approximation Theory	18
Mathematica Slovaca	18
Miskolc Mathematical Notes	18
Nonlinear Dynamics	18
Proceedings IEEE International Symposium on Circuits and Systems	18
Proceedings of The American Mathematical Society	18
Studia Universitatis Babes Bolyai Mathematica	18
Annals of Statistics	17
Discrete Dynamics in Nature and Society	17
Journal of Applied Mathematics and Computing	17
Journal of Intelligent and Fuzzy Systems	17

Axioms	16
Energy	16
Fractals	16

Figure 3 shows the dynamics of the number of mentions of the keyword "Bernstein polynomial" in the four most popular journals from Table 1. The number of mentions of the keyword in Applied Mathematics and Computation has significantly decreased since 2015, while the number of mentions of the keyword "Bernstein polynomial" in the journal Mathematical Methods in the Applied Sciences has increased. Such changes could be related to the increased difficulty of publishing in the mentioned journal, changes in editorial policy or the impact factor, preferences of academics, etc.

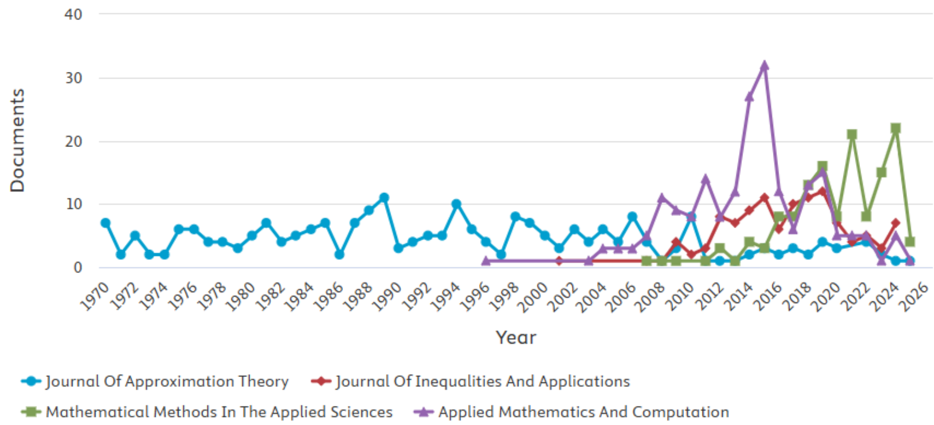


Figure 3. The dynamics of the number of mentions of the keyword "Bernstein polynomial" in the four most popular journals from Table 1.

Table 2 shows the list of authors' countries where documents with the keywords "Bernstein polynomial" were found in one of the search fields. As can be seen, China, USA and India dominate the list. Russia and Libya are mentioned twice in the Scopus database under different names for which the corresponding number of papers has been added. Some papers were not identified by geographical location, but their total share is only 1.3%. Russia's 19th place is surprising, as Bernstein polynomials were discovered there. However, this may be due to the authors' preference to publish their related research in Russian sources that are not indexed in academic databases. E-library (https://elibrary.ru/), a central Russian academic database, mentions only 67 documents for "полином Бернштейна" (singular form), 48 documents for "многочлен Бернштейна" (singular form), 101 documents with "многочлены Бернштейна" (plural form) and 182 documents with "полиномы Бернштейна" (plural form) in all fields, including the full text of a document.

Table 2. The list of authors' countries.

Country/Territory	Documents	Percentage
China	1530	16.43%
United States	1371	14.73%
India	1113	11.95%
Turkey	935	10.04%
Iran	899	9.66%
Romania	509	5.47%
South Korea	508	5.46%
Germany	494	5.31%
Saudi Arabia	389	4.18%
Italy	373	4.01%
France	299	3.21%
Canada	278	2.99%

United Kingdom	268	2.88%
Spain	259	2.78%
Egypt	195	2.09%
Taiwan	188	2.02%
Pakistan	165	1.77%
Japan	161	1.73%
Russia	151	1.62%
Poland	143	1.54%
Malaysia	135	1.45%
Iraq	127	1.36%
Australia	126	1.35%
Jordan	92	0.99%
South Africa	76	0.82%
Switzerland	74	0.79%
Netherlands	71	0.76%
Bulgaria	70	0.75%
Thailand	70	0.75%
Portugal	69	0.74%
Israel	67	0.72%
Hong Kong	65	0.70%
Singapore	65	0.70%
Belgium	64	0.69%
Mexico	64	0.69%
Algeria	60	0.64%
Hungary	54	0.58%
Brazil	50	0.54%
Viet Nam	50	0.54%
Austria	43	0.46%
Greece	43	0.46%
Norway	41	0.44%
United Arab Emirates	40	0.43%
Ukraine	38	0.41%
Morocco	35	0.38%
Nigeria	35	0.38%
Sweden	33	0.35%
Azerbaijan	31	0.33%
Tunisia	29	0.31%
New Zealand	26	0.28%
Serbia	25	0.27%
Chile	23	0.25%
Denmark	23	0.25%
Ireland	22	0.24%
Czech Republic	21	0.23%
Indonesia	20	0.21%
Oman	20	0.21%
Yemen	20	0.21%
Colombia	19	0.20%
Bangladesh	17	0.18%
Finland	16	0.17%
Slovenia	15	0.16%
Lebanon	13	0.14%
Argentina	11	0.12%
Estonia	11	0.12%

Kazakhstan	11	0.12%
Kuwait	11	0.12%
Palestine	10	0.11%
Qatar	10	0.11%
Libya	8	0.09%
Ethiopia	9	0.10%
Cyprus	7	0.08%
Macao	6	0.06%
Slovakia	6	0.06%
Croatia	5	0.05%
Georgia	5	0.05%
Cameroon	4	0.04%
Ecuador	4	0.04%
Lithuania	4	0.04%
Tanzania	4	0.04%
Trinidad and Tobago	4	0.04%
Uruguay	4	0.04%
Uzbekistan	4	0.04%
Venezuela	4	0.04%
Afghanistan	3	0.03%
Bahrain	3	0.03%
Dominican Republic	3	0.03%
Peru	3	0.03%
Philippines	3	0.03%
Belarus	2	0.02%
Cote d'Ivoire	2	0.02%
Ghana	2	0.02%
Kyrgyzstan	2	0.02%
Malawi	2	0.02%
North Korea	2	0.02%
Yugoslavia	2	0.02%
Albania	1	0.01%
Armenia	1	0.01%
Benin	1	0.01%
Chad	1	0.01%
Honduras	1	0.01%
Jamaica	1	0.01%
Kenya	1	0.01%
Luxembourg	1	0.01%
Madagascar	1	0.01%
Mauritius	1	0.01%
Moldova	1	0.01%
Nepal	1	0.01%
North Macedonia	1	0.01%
Sierra Leone	1	0.01%
Swaziland	1	0.01%
Syrian Arab Republic	1	0.01%
Undefined	122	1.31%

Two South Korean universities from Seoul top the list of authors' affiliations, followed by a Romanian, an Indian, a Chinese, a Saudi Arabian and a Turkish university (see Fig. 4).

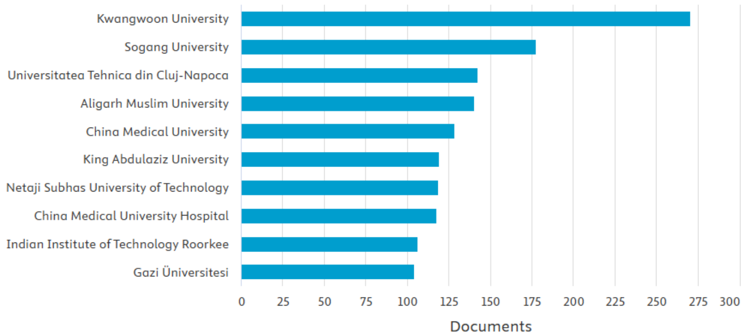
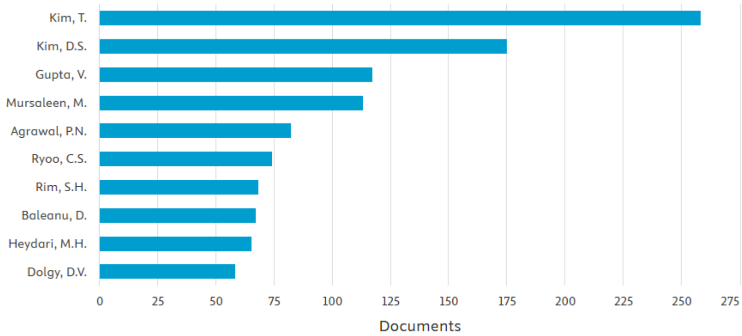


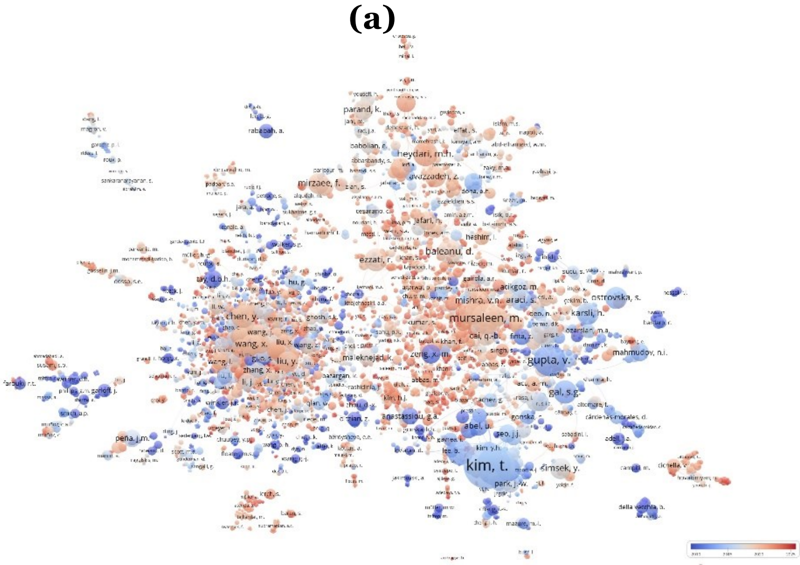
Figure 4. Top ten universities with documents containing the keyword "Bernstein polynomial" in all search fields.

Other universities and institutions mentioned in more than 30 documents include Ankara University (Turkey), Universitatea Babeş-Bolyai (Romania), Zhejiang University (China), Islamic Azad University (Iran), University of Oradea (Romania), Kyungpook National University (South Korea), Eastern Mediterranean University (Cyprus), Hannam University (South Korea), Akdeniz University (Turkey), French National Centre for Scientific Research (France), Shiraz University of Technology (Iran), University of Zaragoza (Spain), Çankaya University (Turkey), Ministry of Education of the People's Republic of China, Islamic Azad University, Karaj Branch (Iran), Shahid Beheshti University (Iran), Lucian Blaga University of Sibiu (Romania), Institute for Space Sciences, Bucharest (Romania), Pukyong National University (South Korea), Xiamen University (China), Malayer University (Iran), Atilim University (Turkey), Gaziantep University (Turkey), Bolu Abant İzzet Baysal University (Turkey).

Figure 5 shows the top ten academics and the co-authorship network overlay visualisation for 2010-2025 (the colours change from blue for older documents to red for newer documents).



(a)



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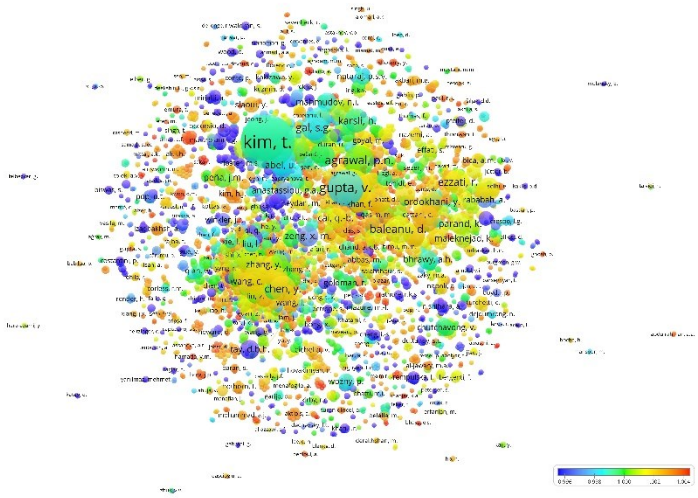

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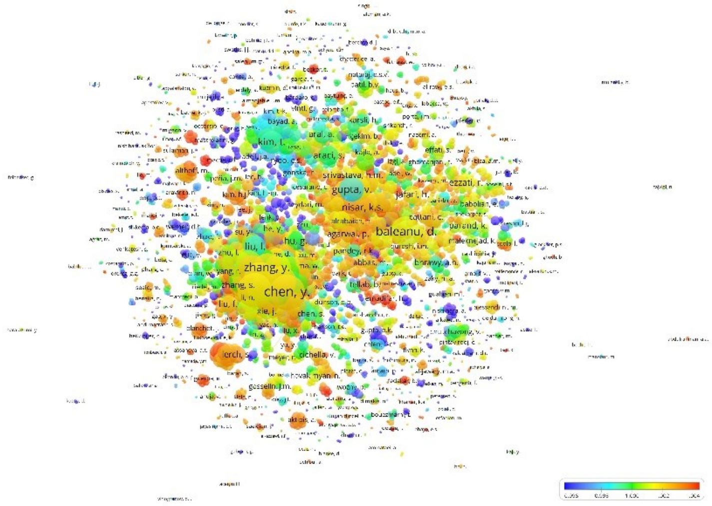

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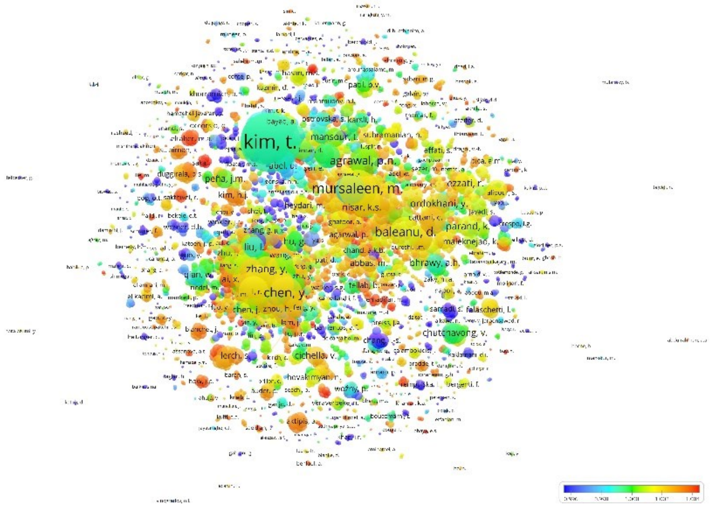


(e)

Figure 5. (a) Top ten academics whose documents contain the keyword "Bernstein polynomial" in all search fields, (b) Co-authorship overlay visualisation with a more dynamic variant available online at https://youtu.be/ipblMZ_saKA **(c)** Overlay visualisation in which the size of each node represents the weight based on the number of documents, while the colour,

which changes from blue to red, reflects the normalised average publication year[5], with blue representing older documents and red representing more recent ones. **(d)** Overlay visualisation in which the size of each node represents the weight based on the number of links, while the colour, which changes from blue to red, reflects the normalised average publication year, with blue representing older documents and red representing more recent ones. **(e)** Overlay visualisation in which the size of each node represents the weight based on the total link strength, while the colour, which changes from blue to red, reflects the normalised average publication year, with blue representing older documents and red representing more recent ones.

The list of funding agencies is shown in Table 2. The leading funding agency is the National Natural Science Foundation of China, and the third one is the Ministry of Science and Technology of the People's Republic of China, which indicates the enormous support provided by China for research in areas related to Bernstein polynomials and their applications. Although the National Research Foundation of Korea is mentioned as a funding agency in only 68 documents, authors from this country are among the most active researchers mentioning the "Bernstein polynomial" in their documents (Fig. 5a).

Table 2. The list of funding sponsors mentioned in documents with the keywords "Bernstein polynomial" was found in any search field. Unfortunately, some funding agencies do not include the name of a country in their title, and some are written in a language other than English.

Funding sponsor	Documents
National Natural Science Foundation of China	657
National Science Foundation	293
Ministry of Science and Technology of the People's Republic of China	209
European Commission	94
Natural Sciences and Engineering Research Council of Canada	88
National Research Foundation of Korea	68
Deutsche Forschungsgemeinschaft[6]	67
National Institutes of Health	62
U.S. Department of Defense	62
Japan Society for the Promotion of Science	56
Fundamental Research Funds for the Central Universities	54
University Grants Commission	53
Office of Naval Research	49
National Key Research and Development Program of China	47
Department of Science and Technology, Ministry of Science and Technology, India	46
Ministry of Education of the People's Republic of China	42
Natural Science Foundation of Fujian Province	41
Science and Engineering Research Board	38
European Regional Development Fund	37
Ministerio de Economía y Competitividad[7]	37

[5] According to the VOSviewer manual available online (https://www.vosviewer.com/documentation/Manual_VOSviewer_1.6.20.pdf), "Avg. pub. year" means the average publication year of documents containing a keyword or term, or the average publication year of documents published by a source, author, organisation or country.

[6] German Research Foundation

[7] Ministry of Economy and Competitiveness, Spain

Air Force Office of Scientific Research	36
Council of Scientific and Industrial Research, India	35
U.S. Department of Energy	35
U.S. Department of Health and Human Services	35
Seventh Framework Programme	34
Engineering and Physical Sciences Research Council	33
Government of Canada	33
Natural Science Foundation of Zhejiang Province	33
Schweizerischer Nationalfonds zur Förderung der Wissenschaftlichen Forschung[8]	33
European Research Council	32
China Postdoctoral Science Foundation	31
Horizon 2020 Framework Programme	31
Ministry of Education, Culture, Sports, Science and Technology	31
Natural Science Foundation of Hebei Province	31
Ministry of Human Resource Development	30
Akdeniz University, Turkey	29
Directorate for Mathematical and Physical Sciences	29
U.S. Air Force	28
Air Force Materiel Command	27
Ministerio de Ciencia, Innovación y Universidades[9]	26
Ministero dell'Istruzione, dell'Università e della Ricerca[10]	26
U.S. Navy	26
Natural Science Foundation of Jiangsu Province	24
UK Research and Innovation	23
Istituto Nazionale di Alta Matematica "Francesco Severi"[11]	22
Junta de Andalucía[12]	22
National Aeronautics and Space Administration	22
Agencia Estatal de Investigación[13]	21
China Scholarship Council	21
Fundação para a Ciência e a Tecnologia[14]	21

The second source of funding mentioned in 293 documents, the National Science Foundation (NSF), doesn't mention any country by name, but an analysis of the locations of the universities shows that 75.32% of all workplaces are in US universities, and 5.06% are Chinese, 2.74% are Canadian, 2.11% are German and British, 1.69% are French, and so on. Therefore, it's evident that the National Science Foundation (https://nsf.gov) is an independent agency of the United States federal government that supports fundamental research and education. However, for efficient tracking of research funding, the name of the fund mentioned in manuscripts should be unique, and various academic databases can also integrate these names into the document properties so that it would be easy to know that the development of a document was supported by research funding.

Analysis of author names in NSF-supported documents using AI chatbots[15] revealed that some US-based academics have names of Chinese and other origins (Table 3), but different tools produced different results.

[8] Swiss National Science Foundation, Switzerland
[9] Ministry of Science, Innovation and Universities, Spain
[10] Ministry of Education, University and Research, Italy
[11] The Francesco Severi National Institute of Higher Mathematics, Italy
[12] Regional Government of Andalusia, Spain
[13] State Research Agency, Spain
[14] The Foundation for Science and Technology, Portugal

Table 3. AI-based analysis of author names using different LLMs performed on 23 January 2025. The values indicate the percentage of author names that each model assigned to a given country, i.e. the proportional distribution of inferred origins. The 'Average' column shows the mean percentage across the three models.

Name origin	Llama 3	Perplexity	Mixtral	Average
United States	34.86%	35.00%	30.60%	33.49%
China	20.41%	20.00%	14.60%	18.34%
United Kingdom	5.36%	8.00%	2.40%	5.25%
Germany	3.59%	5.00%	6.10%	4.90%
Canada	3.59%	5.00%	7.90%	5.50%
France	3.59%	4.00%	4.30%	3.96%
India	3.59%	3.00%	5.50%	4.03%
Japan	1.79%	2.00%	0.60%	1.46%
Others	23.22%	18.00%	28.00%	23.07%
Total	100.00%	100.00%	100.00%	

Table 4 shows the top 10 most cited papers, including citation counts, journal names, and publishers. 40% of these top papers were published in Computer Aided Geometric Design, a key Elsevier journal for research in the mathematical foundations of free-form curves, surfaces and solids. A visualisation of the evolution of sources that published ten or more documents related to the keyword "Bernstein polynomial" in data extracted from Scopus can be found online at https://youtu.be/CTZv1iTT7gA.

Table 4. The top 10 most cited articles that contain the keyword "Bernstein polynomial" in the article title, abstract or keywords according to the Scopus database. The rationale behind searching the article title, abstract or keywords rather than all fields was to identify papers that focus on investigating Bernstein polynomials. The article by Sederberg & Parry is indexed twice in Scopus, showing a different number of citations.

Authors	Year	Title	Source title	Cited by	Publisher
Sederberg Thomas W.; Parry Scott R. [7]	1986	Free-form deformation of solid geometric models	Computer Graphics (ACM)	2285	Association for Computing Machinery
Farin G. [8]	1986	Triangular Bernstein-Bézier patches	Computer Aided Geometric Design	497	Elsevier
Farouki R.T. [9]	2012	The Bernstein polynomial basis: A centennial retrospective	Computer Aided Geometric Design	379	Elsevier
Farouki R.T.; Rajan V.T. [10]	1988	Algorithms for polynomials in Bernstein form	Computer Aided Geometric Design	257	Elsevier
Farouki R.T.; Rajan V.T. [11]	1987	On the numerical condition of polynomials in Bernstein form	Computer Aided Geometric Design	241	Elsevier
King J.P. [12]	2003	Positive linear operators which preserve x^2	Acta Mathematica Hungarica	236	Hungarian Academy of Sciences
Gao F.; Wu W.; Lin Y.; Shen S. [13]	2018	Online Safe Trajectory Generation for Quadrotors Using Fast	Proceedings - IEEE International Conference on Robotics and Automation	212	Institute of Electrical and Elec-

[15] It is not possible to use AI chatbots for all 9310 documents, as most chatbots are not able to process very long texts.

		Marching Method and Bernstein Basis Polynomial			tronics Engineers Inc.
Sederberg T.W.; Parry S.R. [14]	1986	Free-form deformation of solid geometric models	Proceedings of the 13th Annual Conference on Computer Graphics and Interactive Techniques, SIGGRAPH 1986	193	Association for Computing Machinery
Bhrawy A.H.; Taha T.M.; Machado J.A.T. [15]	2015	A review of operational matrices and spectral techniques for fractional calculus	Nonlinear Dynamics	181	Kluwer Academic Publishers
Ostrovska S. [16]	2003	q-Bernstein polynomials and their iterates	Journal of Approximation Theory	180	Academic Press Inc.

4. Network Visualisations and Analysis

Network visualisations and analysis were performed using VOSviewer (https://www.vosviewer.com/), a freeware tool for constructing and visualising bibliometric networks. The minimum number of occurrences of a keyword was set to 3, and 5087 keywords met the threshold. VOSviewer has created 21 clusters that can be seen in different colours in Fig. 6a, 161,307 links, with a total link strength of 261,230. Three types of network visualisations are shown in Fig. 6.

In VOSviewer [17-19], we used the following metrics:

- ***Links*** refer to the number of direct connections or relationships between nodes in the graph. In social networks, for example, this is like counting the number of friendships between individuals, but without considering the strength of those friendships.
- ***Total link strength*** is the cumulative weight of all links between nodes in the graph. The weights can represent the intensity, frequency or importance of the relationships. For example, a citation network could reflect the number of times two articles have been co-cited.
- ***Occurrences*** refer to the number of times a particular keyword appears in the dataset used to construct the graph. For example, the occurrence of "Bernstein polynomial" indicates how often this term is mentioned in the data, regardless of its connections to other terms.

Figure 6 contains network visualisations for the keywords studied. Fig. 6a is a basic network visualisation where we can observe several clusters coloured by different colours. In VOSviewer, a cluster represents a group of items (such as keywords, authors, or publications) that are closely related to each other and more strongly connected than to items in other clusters. Each cluster is shown in a distinct colour and typically reflects a thematic or topical area within the overall network, indicating shared research focus, collaboration patterns or conceptual similarity. Fig. 6b is a density visualisation (or density map) where the red colour shows the most popular topics based on the number of occurrences, and Fig. 6c is the most interesting because the years from 1949 to 2025 were normalised and connected to the colour map used, so we can observe that keywords in blue colours, such as algorithms, approximation theory, filters, CAD, convexity, image reconstruction, and others were relatively more common for older manuscripts, and motion planning, trajectories, uncertainty, machine learning, weather forecasting, collision avoidance, neural networks, continuous-time systems, and others can be found in very recent documents. This means a shift from traditional mathematical topics to applications and numerical methods.

The appearance of slightly different upper bound values, such as 1.002 (Figure 6c) and 1.004 (Figure 6), in a VOSviewer colour map for normalised values is usually due to rounding and display precision in the software's colour scale rather than an error in the underlying data.

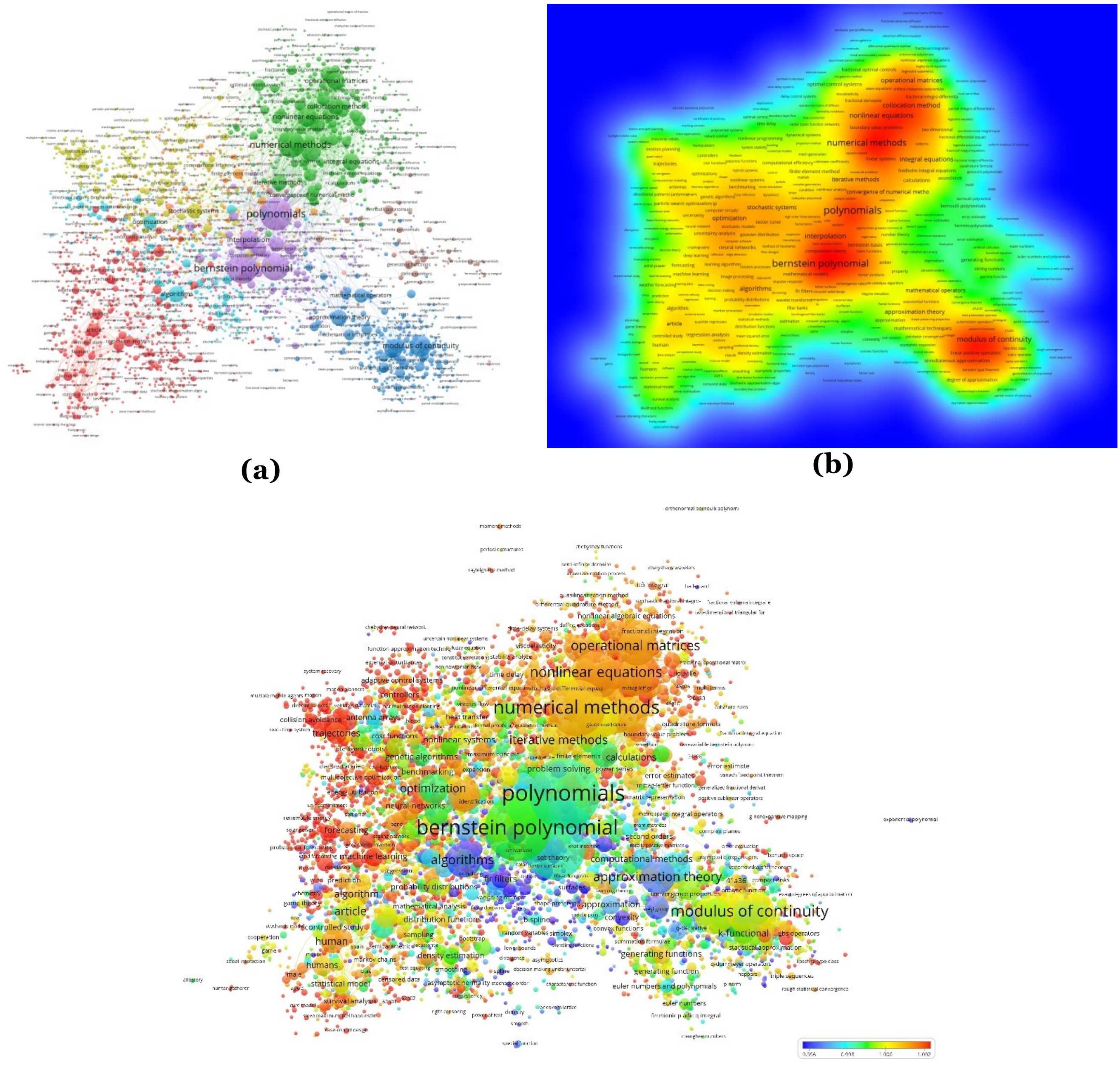
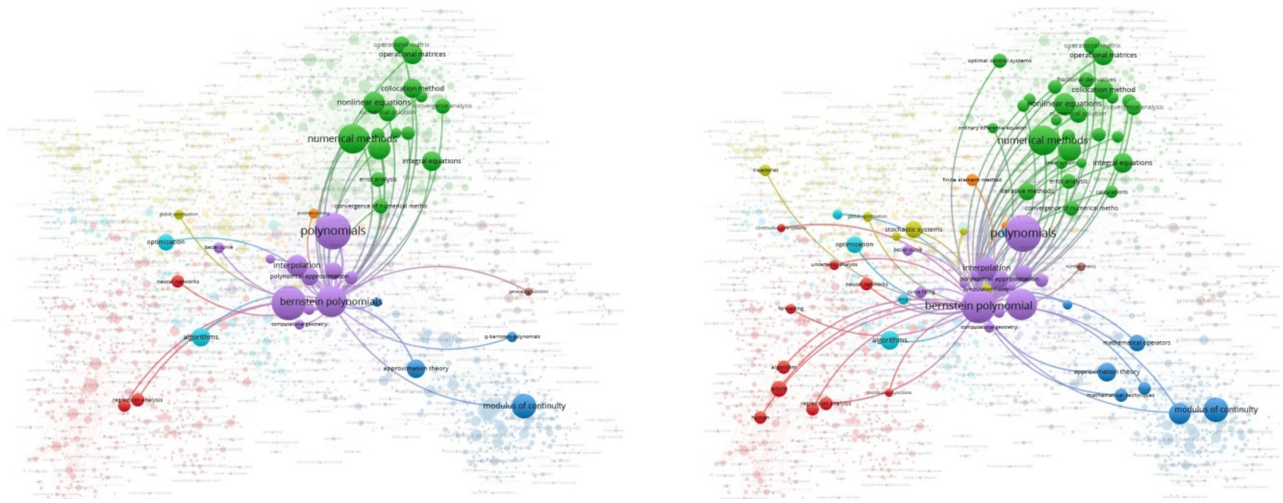


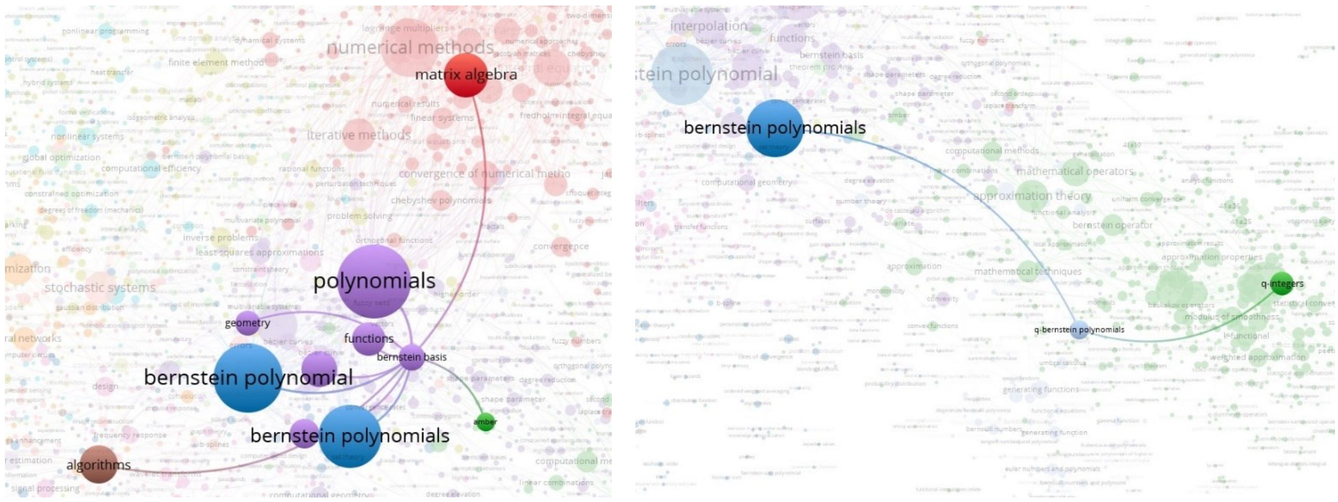
Figure 6. Network visualisation based on the Scopus query and 9310 manuscripts found as of 23 January 2025. **(a)** Network visualisation, **(b)** Density visualisation. A more dynamic visualisation with zooming is available at https://youtu.be/bpqYJVEynQk **(c)** Overlay visualisations where years from 1949 to 2025 are normalised by dividing by the mean. Here, cool colours represent older research topics, while warm colours represent more recent research topics, so the visualisation helps to understand the trends for the related research keywords.

In the network visualisation (Fig. 7), two phrases are related to Bernstein polynomials: the singular and plural forms. Both show stronger links with topics such as trajectories, prediction, optimisation, algorithms, approximation theory, modulus of continuity, mathematical operators, numerical and iterative methods, matrix algebra, operational matrices, etc. For “Bernstein polynomial” (Fig. 7b): Links=2561, Total link strength=7911, Occurrences=888, and for "Bernstein polynomials" (Fig. 7a): Links=1784, Total link strength=4717, Occurrences=728.



(a) **(b)**

Figure 7. Links of (a) "Bernstein polynomials" and (b) "Bernstein polynomials" in the resulting network visualisation. The graph in VOSviewer shows them as different keywords, each with its own links.



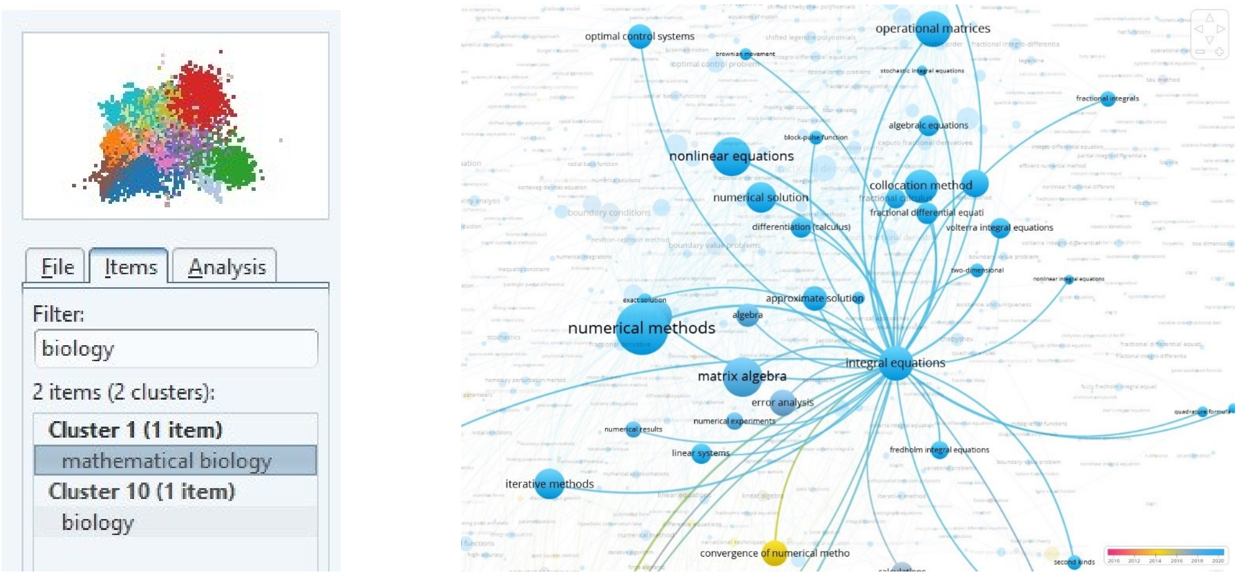
(a) **(b)**

Figure 8. Links of (a) "Bernstein basis" (Links=504, Total link strength=1081) and (b) "*q*-Bernstein polynomial" (Links=162, Total link strength=315) in the resulting network visualisation.

The A-K-J Sankey diagram, created online using the ScienceScape tool, provides a visual representation of the relationship between the main authors (A), keywords (K), and journals (J). The diagram can be accessed via the following link: https://youtu.be/d2hnhklw8io

5. Areas Where Bernstein Polynomials Are Rarely Used

By searching for different keywords in VOSviewer, we can find areas not well connected with Bernstein polynomials in documents from the Scopus database. For example, searching for "biology" returned two items from two clusters (Fig. 9).



(a) (b)

Figure 9. (a) One of the interesting branches of applied mathematics, "mathematical biology" (Links=35, Total link strength=41, Occurrences=3), doesn't have a strong connection with "Bernstein polynomial". We have something similar for “biology” Links=38, Total link strength=39, Occurrences=3. (b) The item "mathematical biology" has a relatively smaller scale than "integral equations", a mathematical tool used to construct mathematical models, particularly in biology and ecology.

A Scopus search for (TITLE-ABS-KEY("Bernstein polynomial") AND TITLE-ABS-KEY("mathematical biology")) returned only four articles, of which only two papers [20, 21] have titles related to mathematical biology. Examples of other keywords are given in Table 5. Strangely, not only "Bernstein polynomial" but both "Bézier curve" and "Bézier surface" appear together with "aerospace engineering" in only fifteen articles. However, the numbers can increase significantly when searching in all fields.

Table 5. Keywords rarely appear with "Bernstein polynomial" in document titles, abstracts or keywords in the Scopus database.

Keyword searched with "Bernstein polynomial" in Scopus in document title, abstract or keywords[16]	Number of documents found	Notes
robotics	21	Only nine articles contain the word "robot" in their title
aerospace engineering	4	Only two articles are related to aerodynamics and flight vehicle
composite material	4	Only two articles mention composites in the title
fuzzy logic	6	Four articles contain the word "fuzzy" in the title
electrical engineering	3	Only one work be-

[16] Academics themselves can try to find such a field or a topic not well related to Bernstein polynomials by using a Scopus query (TITLE-ABS-KEY("Bernstein polynomial") AND TITLE-ABS-KEY("topic of your interest")), or by using "Bernstein polynomial" AND "topic of your interest" in Scholar Google.

		longs to a mathematical journal
energy engineering	0	No documents were found
nuclear	3	Two documents published in bioscience sources
mechanical engineering	1	Recent work published in 2023
special function[17]	9	Four articles were published in 2024

A chat has been started with AI chatbots about the possible applications of Bernstein polynomials in certain sub-fields of engineering and mathematics mentioned as keywords in Tables 5 and 7. Interested readers can check out the links in Table 6.

Table 6. Suggestions from various AI chatbots about applications of Bernstein polynomials in different fields.

AI chatbot	Responses
ChatGPT	https://bit.ly/3EjGUlu
Google Gemini	https://bit.ly/3PX0Htg
Perplexity	https://bit.ly/3Ei5N0Q
Microsoft Copilot	https://bit.ly/42B1R5u

More detailed Table 7 shows the co-occurrence of a given keyword related to the Mathematical Subjects Classification and the keyword "Bernstein polynomial" in documents in the Scopus database. These numbers have been manually extracted from the Scopus database. The information can be helpful for academics looking for new or underexplored areas where Bernstein polynomials can be applied. Interestingly, the Pearson correlation coefficient between the number of documents found by searching in "All fields" and "Article title, abstract, keywords" equals 0.848, indicating a powerful and positive relationship between the two variables.

Table 7. Co-occurrence of a given keyword(s) related to the Mathematical Subjects Classification and "Bernstein polynomial" in documents in the Scopus database. The two columns "All fields" and "Article title, abstract, keywords" use the same colour map (Green-Yellow-Red scale in conditional formatting in Microsoft Excel) but are applied separately to each other, where red colour indicates low values, and green indicates high values (number of documents).

Top-level code	Mathematics Subject Classification (MSC) and used keywords (in " ")	All fields	Article title, abstract, keywords
00	General		
	"recreational mathematics"	0	0
	"philosophy of mathematics"	0	0
	"mathematical modelling" OR "mathematical modelling"	1291	9
01	History and biography		
	"history" and "biography"	7	0
03	Mathematical logic and foundations		
	"mathematical logic"	4	0
05	Combinatorics		
	"combinatorics"	248	2

[17] This smaller number of documents doesn't mean that Bernstein polynomials or their generalisations cannot be represented as a special case of more general special functions or in terms of other polynomials, such as Jacobi polynomials [22].

06	Order, lattices, ordered algebraic structures		
	"lattice"	277	12
	"ordered algebraic structure"	0	0
08	General algebraic systems		
	"algebraic system"	115	16
11	Number theory		
	"number theory"	383	25
12	Field theory and polynomials		
	"field theory"	60	0
13	Commutative algebra (Commutative rings and algebras)		
	"commutative algebra"	20	0
	"commutative ring"	11	0
14	Algebraic geometry		
	"algebraic geometry"	108	1
15	Linear and multilinear algebra; matrix theory		
	"linear algebra"	348	13
	"multilinear algebra"	35	0
	"matrix theory"	29	0
16	Associative rings and (associative) algebras		
	"associative ring"	0	0
	"associative algebra"	1	0
17	Non-associative rings and (non-associative) algebras		
	"non-associative ring"	0	0
	"non-associative algebra"	0	0
18	Category theory; homological algebra		
	"category theory"	2	0
	"homological algebra"	4	0
19	K-theory		
	"K-theory"	4	0
20	Group theory and generalisations		
	"group theory"	33	0
22	Topological groups, Lie groups (and analysis upon them)		
	"topological group"	2	0
	"Lie group"	65	1
26	Real functions (including derivatives and integrals)		
	"real function"	77	8
28	Measure and integration		
	"integration"	1703	107
30	Functions of a complex variable (including approximation theory in the complex domain)		
	"complex variable"	114	2
	"complex domain"	79	2
	"approximation theory"	2788	98
31	Potential theory		
	"potential theory"	51	1
32	Several complex variables and analytic spaces		
	"several complex variables"	15	0
	"analytic space"	9	0
33	Special functions		
	"special function"	776	9

34	Ordinary differential equations		
	"ordinary differential equation"	539	39
	"ODE"	219	15
35	Partial differential equations		
	"partial differential equation"	1351	62
	"PDE"	137	8
37	Dynamical systems and ergodic theory		
	"dynamical system"	558	17
	"ergodic theory"	13	1
39	Difference (equations) and functional equations		
	"difference equation"	1227	5
	"functional equation"	182	20
40	Sequences, series, summability		
	"sequence"	1668	161
	"series"	4080	130
	"summability"	280	18
41	Approximations and expansions		
	"approximation"	6026	765
	"expansion"	1268	107
42	Harmonic analysis on Euclidean spaces (including Fourier analysis, Fourier transforms, trigonometric approximation, trigonometric interpolation, and orthogonal functions)		
	"harmonic analysis"	186	3
	"Fourier analysis"	241	5
	"Fourier transform"	187	9
	"trigonometric approximation"	54	2
	"trigonometric interpolation"	16	2
	"orthogonal function"	194	14
43	Abstract harmonic analysis		
	"abstract harmonic analysis"	1	0
44	Integral transforms, operational calculus		
	"integral transform"	370	1
	"operational calculus"	30	0
45	Integral equations		
	"integral equation"	1473	138
46	Functional analysis (including infinite-dimensional holomorphy, integral transforms in distribution spaces)		
	"functional analysis"	1017	14
	"infinite-dimensional holomorphy"	0	0
	"integral transform in distribution space"	0	0
47	Operator theory		
	"operator theory"	552	5
49	Calculus of variations and optimal control; optimisation (including geometric integration theory)		
	"calculus of variations"	201	4
	"optimal control"	812	48
	"optimization" or "optimisation"	2585	237
	"geometric integration theory"	3	0
51	Geometry		
	"geometry"	958	124
52	Convex (geometry) and discrete geometry		

	"convex geometry"	6	0
	"discrete geometry"	3	0
53	Differential geometry		
	"differential geometry"	79	2
54	General topology		
	"general topology"	22	0
55	Algebraic topology		
	"algebraic topology"	7	0
57	Manifolds and cell complexes		
	"manifold"	201	9
	"cell complex"	1	0
58	Global analysis, analysis on manifolds (including infinite-dimensional holomorphy)		
	"global analysis"	27	0
	"analysis on manifolds"	8	0
60	Probability theory and stochastic processes		
	"probability theory"	313	8
	"stochastic process"	284	11
62	Statistics		
	"statistics"	2223	89
65	Numerical analysis		
	"numerical analysis"	1465	23
68	Computer science		
	"computer science"	1841	6
70	Mechanics of particles and systems (including particle mechanics)		
	"mechanics of particles"	0	0
	"mechanics of systems"	0	0
74	Mechanics of deformable solids		
	"mechanics of deformable solids"	0	0
76	Fluid mechanics		
	"fluid mechanics"	270	7
78	Optics, electromagnetic theory		
	"optics"	204	3
	"electromagnetic theory"	29	0
80	Classical thermodynamics, heat transfer		
	"thermodynamics"	50	1
	"classical thermodynamics"	0	0
	"heat transfer"	348	14
81	Quantum theory		
	"quantum theory"	25	3
82	Statistical mechanics, structure of matter		
	"statistical mechanics"	432	0
	"structure of matter"	0	0
83	Relativity and gravitational theory (including relativistic mechanics)		
	"relativity theory" or "theory of relativity"	2	0
	"gravitational theory", "theory of gravity", or "theory of gravitation"	2	0
	"relativistic mechanics"	0	0
85	Astronomy and astrophysics		
	"astronomy"	163	1
	"astrophysics"	139	3

86	Geophysics		
	"geophysics"	107	1
90	Operations research, mathematical programming		
	"operations research"	199	0
	"mathematical programming"	205	5
91	Game theory, economics, social and behavioural sciences		
	"game theory"	55	2
	"economics"	508	5
	"social science"	87	2
	"behavioural science"	19	0
92	Biology and other natural sciences		
	"biology"	501	14
	"natural science"	257	3
93	Systems theory; control (including optimal control)		
	"systems theory" or "theory of systems"	140	0
	"control theory"	302	5
	"optimal control"	812	48
94	Information and communication, circuits		
	"information and communication"	49	0
	"circuit"	746	44
97	Mathematics education		
	mathematics education or "math education", or "maths education" or "mathematical education."	244	0

6. Discussion

The bibliometric analysis of published literature is essential for understanding a research field from a bird's-eye view: to understand which topics were popular or unpopular at the time, the significance of the works and authors, the links between them and other fields, and to identify new directions for future research. Unfortunately, not all published academic manuscripts are available in a single academic database such as Scopus. Instead, they are distributed across the internet in many databases, so extracting publication-related data can take a long time. Perhaps one day in the future, we will have a single tool that allows academics to easily access all the knowledge humanity has generated throughout its existence, summarise it quickly, translate it between languages, and use integrated AI agents as intellectual assistants.

7. Conclusions

In this work, we carried out a bibliometric analysis of Scopus data for documents containing the keyword "Bernstein polynomial" across all fields. The analysis showed the main areas of research where Bernstein polynomials are often and rarely used. It also revealed how publication trends and keywords have changed over time, and identified the most cited sources, most mentioned countries, institutions, and most active authors contributing to this topic. The analysis also resulted in various visualisations of keyword networks and author networks.

Our findings show that Bernstein polynomials are frequently applied in approximation theory, optimisation, sequences and series, integral equations, and geometry, while their presence in fields such as quantum theory, mathematical logic, associative algebra, homological algebra, and relativistic mechanics remains very low. Academics from the United States, China, India, Turkey, and Iran were identified as the most active contributors. Citation analysis indicated that the highly cited papers mainly focus on the development and application of Bernstein polynomials and Bézier forms in geometric modelling, numerical analysis, and approximation theory, including algorithms, numerical stability, and modern extensions such as q-Bernstein and fractional calculus methods.

Furthermore, we have found underexplored connections between Bernstein polynomials and fields such as robotics, aerospace engineering, composite materials, and others. These observations suggest promising directions for future research. To explore these possibilities, a dialogue with AI chatbots was initiated, which generated several preliminary suggestions for interdisciplinary applications.

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References

1. Farin, G. Curves and Surfaces for Computer-Aided Geometric Design: A Practical Guide; Elsevier: Amsterdam, The Netherlands, 2014.
2. Lorentz, G.G. Bernstein Polynomials; American Mathematical Society, 2012.
3. Karacı, A.; Büyükyazıcı, İ.; Aktümen, M. Recognition of human speech using q-Bernstein polynomials. Int. J. Comput. Appl. 2010, 975, 8887.
4. Petrone, S. Random Bernstein polynomials. Scand. J. Stat. 1999, 26(3), 373–393.
5. Benavoli, A.; Facchini, A.; Zaffalon, M. Bernstein's socks and polynomial-time provable coherence. arXiv 2019, arXiv:1903.04406.
6. Farouki, R.T. The Bernstein polynomial basis: A centennial retrospective. Comput. Aided Geom. Des. 2012, 29(6), 379–419.
7. Sederberg, T.W.; Parry, S.R. Free-Form Deformation of Solid Geometric Models. Comput. Graph. 1986, 20, 151–160.
8. Farin, G. Triangular Bernstein-Bézier Patches. Comput. Aided Geom. Des. 1986, 3, 83–127.
9. Farouki, R.T. The Bernstein Polynomial Basis: A Centennial Retrospective. Comput. Aided Geom. Des. 2012, 29, 379–419.
10. Farouki, R.T.; Rajan, V.T. Algorithms for Polynomials in Bernstein Form. Comput. Aided Geom. Des. 1988, 5, 257–281.
11. Farouki, R.T.; Rajan, V.T. On the Numerical Condition of Polynomials in Bernstein Form. Comput. Aided Geom. Des. 1987, 4, 241–256.
12. King, J.P. Positive Linear Operators Which Preserve x^2. Acta Math. Hungar. 2003, 99, 203-208.
13. Gao, F.; Wu, W.; Lin, Y.; Shen, S. Online Safe Trajectory Generation for Quadrotors Using Fast Marching Method and Bernstein Basis Polynomial. In Proceedings of the 2018 IEEE International Conference on Robotics and Automation (ICRA), Brisbane, QLD, Australia, 21–25 May 2018; pp. 7034–7040.
14. Sederberg, T.W.; Parry, S.R. Free-Form Deformation of Solid Geometric Models. In Proceedings of the 13th Annual Conference on Computer Graphics and Interactive Techniques (SIGGRAPH 1986), Dallas, TX, USA, 18–22 August 1986; pp. 151–160.
15. Bhrawy, A.H.; Taha, T.M.; Machado, J.A.T. A Review of Operational Matrices and Spectral Techniques for Fractional Calculus. Nonlinear Dyn. 2015, 81, 109–135.
16. Ostrovska, S. *q*-Bernstein Polynomials and Their Iterates. J. Approx. Theory 2003, 123, 232–255.
17. Van Eck, N.; Waltman, L. Software survey: VOSviewer, a computer program for bibliometric mapping. Scientometrics 2010, 84(2), 523–538.
18. Orduña-Malea, E.; Costas, R. Link-based approach to study scientific software usage: The case of VOSviewer. Scientometrics 2021, 126(9), 8153–8186.
19. Bukar, U. A., Sayeed, M. S., Razak, S. F. A., Yogarayan, S., Amodu, O. A., & Mahmood, R. A. R. (2023). A method for analyzing text using VOSviewer. MethodsX, 11, 102339.

20. Venturino, E.; Aniţa, S.; Mezzanotte, D.; Occorsio, D. A High Order Numerical Scheme for a Nonlinear Nonlocal Reaction–Diffusion Model Arising in Population Theory. Journal of Computational and Applied Mathematics 2024, 116082.
21. Yousefi, S.A.; Behroozifar, M.; Dehghan, M. Numerical Solution of the Nonlinear Age-Structured Population Models by Using the Operational Matrices of Bernstein Polynomials. Appl. Math. Model. 2012, 36, 3, 945–963.
22. Soleyman, F.; Area, I.; Masjed-Jamei, M.; Nieto, J.J. Representation of (p,q)-Bernstein polynomials in terms of (p,q)-Jacobi polynomials. J. Inequal. Appl., 2017, 167.